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RAN-2203000205023003

T. Y. B. Sc. (Sem. - V) Examination March - 2023

Mathematics : MTH - 503

Real Analysis - I (New Course)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(1) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:
☛ **T. Y. B. Sc. (Sem. - V)**

Name of the Subject :
☛ **Mathematics : MTH - 503 Real Analysis - I (New Course)**

Subject Code No.: **2203000205023003**

Seat No.:

Student's Signature

(2) Figures to the right indicate marks of the question.
(3) Follow usual notations and conventions.

Q. 1 Answer any FIVE from the following : **[10]**

1. Define bounded sequence of real numbers with an illustration.
2. In usual notations prove that $\lim_{n\to\infty} s_n = L \Rightarrow \lim_{n\to\infty} cs_n = cL$ for $c < 0$.
3. Write statement of "Nested interval theorem".
4. Find limit inferior for the sequence $\left\{\left(1+\frac{1}{n}\right)\right\}_{n=1}^{\infty}$
5. Define convergence and conditional convergence of a series of real numbers.
6. Find $\sum_{n=1}^{\infty} p_n$ and $\sum_{n=1}^{\infty} q_n$ for the series $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{n+1}\right)$.
7. Give statement of the "ROOT TEST" for the absolute convergence of the series of real numbers.
8. Using the concept of "dominated" show that the series $\sum_{n=1}^{\infty}\frac{3}{n}$ is divergent.

Q. 2. **Answer any TWO.** **[10]**

(a) Prove that a nondecreasing sequence which is bounded above is convergent.

(b) Let $s_1 = \sqrt{2}$ and let $s_{n+1} = \sqrt{2} \cdot \sqrt{s_n}$ for $n \geq 2$. Prove that $\{s_n\}_{n=1}^{\infty}$ is convergent.

(c) Define a monotone sequence and show that $\{s_n\}_{n=1}^{\infty}$ is monotone where $s_n = \dfrac{1 \cdot 3 \cdot 5 \ldots \cdot (2n-1)}{2 \cdot 4 \cdot 6 \ldots \cdot 2n}$

Q. 3. **Answer any TWO.** **[10]**

(a) Let $\{s_n\}_{n=1}^{\infty}$ diverges to infinity and if $\{t_n\}_{n=1}^{\infty}$ is bounded then prove that $\{s_n + t_n\}_{n=1}^{\infty}$ diverges to infinity.

(b) Define a Cauchy sequence and if the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ converges, then prove that $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence.

(c) Let $\{a_n\}_{n=1}^{\infty}$ be a sequence of real numbers and for each $n \in I$, let

$s_n = a_1 + a_2 + a_3 + \ldots. + a_n$

$t_n = |a_1| + |a_2| + |a_3| + \ldots. + |a_n|$

Prove that if $\{t_n\}_{n=1}^{\infty}$ is a Cauchy sequence, then so is $\{s_n\}_{n=1}^{\infty}$.

Q. 4. **Answer any TWO.** **[10]**

(a) Stat and prove the Leibnitz test for the convergence of an alternating series.

(b) Classify the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots.$ as to divergent, conditionally convergent, or absolutely convergent.

(c) Discuss the convergence of the series $\sum_{n=1}^{\infty} \log\left(1 + \frac{1}{n}\right)$.

Q. 5. **Answer any TWO.** **[10]**

(a) If $\{a_n\}_{n=1}^{\infty}$ is a non-increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} 2^n a_{2^n}$ converges then prove that $\sum_{n=1}^{\infty} a_n$ converges.

(b) Using appropriate test of convergence check the convergence for the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$.

(c) For what values of x does the series $\sum_{n=1}^{\infty} \frac{1}{n^x}$ converge?
